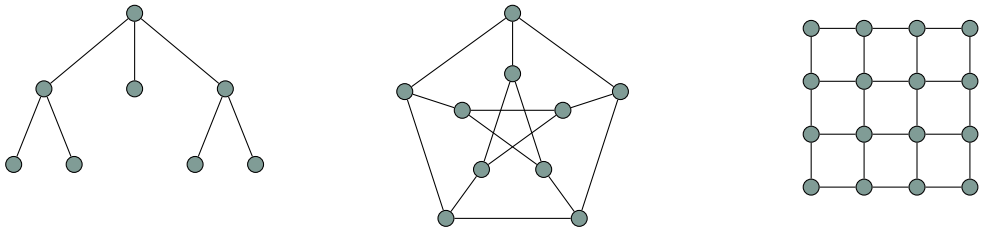
Math 107: Proof guidelines



1. Set up the proof properly.
   - State what you are assuming and what you plan to prove.
     - *Let G be a graph and assume that G has minimum degree at least two. We will show that G contains a cycle.*
   - Are you using proof by contradiction? Induction? A minimality assumption? Understand what your proof structure is before you start writing.
   - Make sure that the proof set-up is correct and is clearly explained at the start of the proof.
     - *Assume for a contradiction that G does not contain a cycle. Therefore, G is a forest.*
   - If you are using any non-standard notation, be sure to clearly state what the notation means.
2. Explain each step in the proof.
   - Every logical step should have a clearly-explained justification. Is it because of a definition? A theorem? A counting argument? Someone should be able to understand why each sentence holds from reading what you have written.
     - *Choose a connected component C of G. Since C is connected and acyclic, C is a tree. Furthermore, since G has minimum degree at least two, C has at least three vertices.*
   - Definitions and theorems from class or the course notes may be used by name.
     - *Every tree with more than one vertex has at least one leaf, so C has a leaf v.*

- If you use a result in your proof that is not from class or the course notes, clearly state and prove the result you are using.
- If you are using induction, make sure you clearly state the base case and clearly state where you use the inductive hypothesis.

3. Conclude the proof.
   - Explain how you have shown the desired statement.
   - If you are using proof by contradiction, state precisely what is contradicted.
     - *Now, v is a vertex of G of degree one, contradicting that G has minimum degree at least two.*
   - Often it is nice to explicitly state, "This proves that..."
     - *This proves that every graph with minimum degree at least two contains a cycle.*
   - To double-check that your proof is correct, make sure that you used every assumption. If you didn't, why not? Was it unnecessary? Is the statement true without the assumption? If the statement is not true without the assumption, why not? Where do you need the assumption?