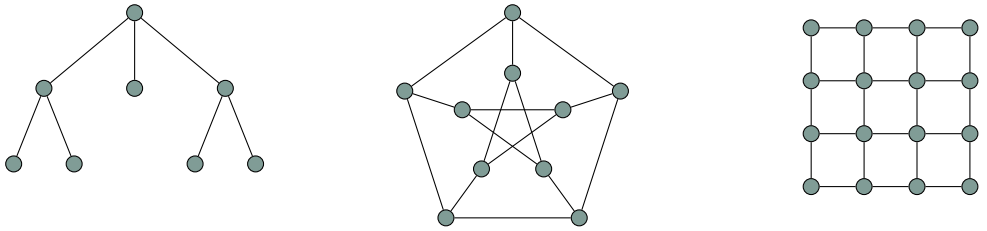
Math 107: Introduction to Graph Theory

Optional Project



1. Overview

The four-color theorem is one of the most famous results in graph theory. It states that every planar graph can be colored with at most four colors. We will discuss planar graphs and graph coloring later in this course, and we will mention the four-color theorem, but we won't do the proof. There's a good reason for this: the proof of the four-color theorem, by Appel and Haken in 1976, uses a computer to check hundreds of cases.

The four-color theorem has an interesting history. In particular, back in 1879, Kempe published a proof of the four-color theorem. Mathematicians accepted this proof for a long time, until, over a decade later, another mathematician named Heawood discovered a mistake in the reasoning. Kempe's proof of the four-color theorem was false, but his ideas still gave a proof of a five-color theorem, and elements of his ideas are used in the correct proof of the four-color theorem.

2. The Project

The project is to write a paper explaining the false proof of the four-color theorem, including why it's false and how it can be corrected to give a five-color theorem. The paper should focus on explaining concepts in a clear and understandable way, and you can think of the intended audience as you at this point in the term: familiar with basic graph theory and curious about the four-color theorem. This should be a fun paper about graph theory, the kind of paper you would show your friends. You can optionally choose to include other content. For example:

- You could include more historical details, such as the reactions of mathematicians when they discovered that a proof they had accepted was in fact wrong or the reason why the four-color theorem was studied to begin with.
- You could include some reflections on the philosophical differences between a human proof of a five-color theorem vs. a computer proof of a four-color theorem.
- You could include some reflections on what it means for mathematics, and for the notions of proof and truth, that a false proof could be accepted for over a decade.

. Those are all optional: the core of the project is to explain Kempe's false proof in a way that you find engaging and enlightening.

3. How it works

I will schedule one or two sessions to meet outside of class with those who want to participate in the optional project. During these sessions, we will go over Kempe's proof and ideas together. There will be plenty of opportunities to ask questions and to make sure you understand the false proof and its five-color fix.

We will also schedule one session later in the term for peer feedback on drafts of the papers.

4. Rubric and grading

The point of this project is to give you an opportunity to explore an idea in graph theory more deeply — in particular, an idea that has interesting history and raises interesting questions about the nature of doing mathematics. It will be worth a small amount of extra credit for your course grade. However, **if your primary motivation is to improve your course grade, please come talk to me instead**: there are better ways to achieve that aim than this project, and I'm happy to help you with that.

The target length of the paper is around 4-6 pages, though the page count is flexible and less important than the quality of the writing. The rubric is based on the following criteria:

(1) Engage in the extra sessions to learn the ideas (we will schedule a time when everyone can make it)

(2) Participate in the draft feedback session by having a draft ready to share and by giving feedback on a classmate's draft

(3) The paper itself should:
   (a) Outline and explain the false proof of the four-color theorem in enough detail that all of the steps make sense
   (b) Explain why the proof is false
   (c) Explain why replacing "four colors" with "five colors" is a fix for the problem

(d) Include an introduction that explains your goal for this paper: what do you hope a reader of your paper will get out of learning about the false proof of the four-color theorem?

Email me or ask after class if you have any questions about this project! Also, **please email me by Monday, January 26th** if you would like to participate in the project.